Math 155, *Lecture Notes- Bonds* **Name_____**

Section 8.1 Basic Integration Rules

Basic Integration Rules ($a > 0$)

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u\,du = e^u + C$
7. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
8. $\int \sin u\,du = -\cos u + C$
9. $\int \cos u\,du = \sin u + C$
10. $\int \tan u\,du = -\ln|\cos u| + C$
11. $\int \cot u\,du = \ln|\sin u| + C$
12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
13. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u\,du = \tan u + C$
15. $\int \csc^2 u\,du = -\cot u + C$
16. $\int \sec u \tan u\,du = \sec u + C$
17. $\int \csc u \cot u\,du = -\csc u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

Procedures for Fitting Integrands to Basic Rules

Technique	*Example*
Expand (numerator).	$(1 + e^x)^2 = 1 + 2e^x + e^{2x}$
Separate numerator.	$\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$
Complete the square.	$\frac{1}{\sqrt{2x - x^2}} = \frac{1}{\sqrt{1 - (x-1)^2}}$
Divide improper rational function.	$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$
Add and subtract terms in numerator.	$\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{(x+1)^2}$
Use trigonometric identities.	$\cot^2 x = \csc^2 x - 1$
Multiply and divide by Pythagorean conjugate.	$\frac{1}{1+\sin x} = \left(\frac{1}{1+\sin x}\right)\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{1-\sin x}{1-\sin^2 x} = \frac{1-\sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos^2 x}$

Ex.1 Integrate: $\int \frac{2x}{x-4} dx$

Ex.2 Evaluate: $\int \sec(3x)\tan(3x)\, dx$

Ex.3 Evaluate: $\int \csc^2(x) e^{\cot(x)} dx$

Ex.4 Evaluate: $\int \frac{5}{3e^x - 2} dx$

Ex.5 Integrate: $\int \frac{1}{\cos(\theta)-1} d\theta$

Ex.6 Evaluate: $\int \frac{2}{3\left(\sec(x)-1\right)}dx$

Ex.7 Integrate: $\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}dx$

Ex.8 Evaluate: $\int_1^e \frac{1-\ln(x)}{x}dx$